Flight Control and Scheduling for the Lateral Motion of a Tailless F-16 using Eigenstructure Assignment

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Some special thanks I have to Ralph Paul whose Diplomarbeite was the basic for the model of the tailless F-16 that was used.

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Abstract

When one studies the history of aircraft design, one finds several attempts to design and build an aircraft with reduced tail size. Although this reduction has sometimes resulted in smaller drag and weight, most of these “tailless” aircraft had severe controllability problems, since the lateral-handling qualities were fairly poor. Regardless of these attempts, the interest in the development of aircraft with reduced tail size — compared with conventional aircraft — has recently increased. This interest is based on the intention of designing future aircraft by using more stealth technology in order to minimize their radar signature. One step to doing this is the reduction of the vertical tail which generates a large signature. The removal of parts of the vertical tail results in a lateral-directional behavior of the aircraft, which is far different than that of a conventional one. In order to get a new control on the lateral axis, a thrust vector control was added to the aircraft. It is obvious that without a reasonable controller the “tailless” aircraft cannot be handled by a pilot.

In this study, the method of the eigenstructure assignment will primarily be used to determine a controller for the “tailless” F-16. Using this method it is possible to design a simple closed-loop controller with a gain matrix in the feed-back loop. The eigenstructure assignment requires a desired system whose eigenstructure is taken to compute the gain matrix of the closed-loop system. A desired model will be developed which allows the placing of three of the four characteristic lateral motion eigenvalues of an aircraft — roll subsidence and two conjugate complex Dutch roll poles — on exact locations in the complex plane. At two different flight conditions, these parameters are varied to determine a controller which yields a advantageous behavior of the aircraft for a simulated roll rate and side slip. Following this, the desired model will be extended by actuator dynamics and
saturation. It will turn out that the parameters have to be chosen in a different way to prevent the system from becoming unstable. The simulation of the system will also reveal that there are some restrictions for the use of the developed controller. These refer primarily on limitations on the step size of the commanded impulse.

By using the results and the knowledge of the previously considered flight conditions — these include a low velocity and a high velocity operating point — a gain scheduling for intermediate velocities will be developed. This scheduling will be evaluated by simulating the response for the determined gain matrix for several velocities.

At the end of this study an LQ controller will be designed in order to see if this kind of controller is capable of yielding comparable results to the eigenstructure assignment. It will be observed that the closed-loop system responds in a different way to stabilize the aircraft after a commanded impulse. Furthermore, it will indicate that it is a problem to command a side slip and a roll rate as directly as for the previous system. One may attempt to improve this by introducing an asymptotic model following — without a basic improvement of the problem, as can already be inferred.
# Table Of Contents

Table Of Figures ........................................................................................................ 7  
Table Of Variables And Acronyms ............................................................................ 11  

1. The Tailless F-16 Model ......................................................................................... 13  
   1.1 Structure Of The Original Non-Linear F-16 Model ........................................ 13  
   1.1.1 Description And Scope Of The Application Of The Dynamic System ...... 13  
   1.1.2 The Non-Linear Model ............................................................................. 14  
   1.2 The Tailless F-16 Model .................................................................................. 19  
   1.2.1 Basic Ideas Behind The Tailless Modifications ....................................... 19  
   1.2.2 Thrust Vector Augmentation ................................................................... 20  
   1.2.3 Tailless F-16 Model Data ........................................................................ 21  
   1.2.4 Lateral Linear Tailless Model .................................................................. 21  
1.3 Lateral Motion Analysis Of The Tailless Model .............................................. 22  
   1.3.1 Evaluation Of The Stability Of An Aircraft ............................................ 22  
   1.3.2 Stability Of The Tailless F-16 ................................................................. 23  

2. Introduction Of The Eigenstructure Assignment ............................................... 27  
   2.1 Specification Of The Control Problem ....................................................... 27  
   2.2 Solution Of The Control Problem .............................................................. 30  

3. Applying The Eigenstructure Assignment For A Lateral Control Design At Different Operating Points ............................................................... 33  
   3.1 Design Of A Desired Model For The Eigenstructure Assignment .................. 33  
   3.2 Evaluation Of The Designed Desired Model For The Case Of A High Velocity Flight Condition ................................................................. 36  
   3.2.1 Linearized Lateral Open-Loop Dynamics Of The Tailless F-16 ............... 36  
   3.2.2 Defaults And Results Of The Eigenstructure Assignment ..................... 37  
   3.2.3 Comparison Of The Transfer Functions Of The Desired System And The Achieved System ............................................................. 38  
   3.2.4 Simulation Of Both Systems With A Commanded Impulse .................... 41  
   3.2.5 Extension Of The Dynamic System By Actuator Dynamics And Saturation ................................................................................................. 44  
   3.2.6 Connection Between The Dutch Roll Pole Position And The System Response For A Commanded Roll Rate .......................................... 51
3.2.7 Restrictions To A Commanded Side Slip ..................................54

3.3 Study Of A Low Velocity Flight Condition ..................................55

3.3.1 Linearized Lateral Open-Loop System And Result Of The Eigenstructure Assignment ..................................55

3.3.2 Simulating The System At The Low Velocity Flight Condition ....57

3.3.3 Extension Of The Original Desired Model ..................................58

3.3.4 Simulation Of A Commanded Roll Rate Using The Extended Model ..................................60

3.3.5 Limits Of The Method For Low Velocities ..................................69

3.3.6 Simulation Of A Commanded Side Slip Using The Extended Model ..................................70

3.3.7 Results Of A Simulation With The Extended Model For A Flight Condition At A High Velocity .................................73

3.4 Evaluation Of The Eigenstructure Assignment As A Way To Design A Controller For The Tailless F-16 .................................74

4. Approach For A Gain Scheduling Of The Closed-Loop System Of The Tailless F-16 ..................................76

4.1 Preparations For The Gain Scheduling ..................................76

4.1.1 Development Of A Method To Compute The Gain Matrix As A Function Of The Velocity ..................................76

4.1.2 Entries Of The Transformation Matrix H ..................................79

4.2 Simulation Of The Closed-Loop System Using The Scheduled Matrices ..................................81

4.2.1 Representation Of The Results Of The Simulations And Used Simplifications For The Scheduling ..................................81

4.2.2 Results Of A Gain Scheduling For A Commanded Roll Rate ....82

4.2.3 Results Of A Gain Scheduling For A Commanded Side Slip ....84

4.3 Study Of The Determined Gain Scheduling For Particular Intermediate Velocities ..................................87

4.3.1 Particular Velocities For A Commanded Roll Rate ..........................87

4.3.2 Particular Velocities For A Commanded Side Slip ..........................90

4.4 Evaluation Of The Results Achieved With The Determined Gain Scheduling ..................................93

5. Development And Evaluation Of A Controller For The Tailless Aircraft Using LQ-Design ..................................94

5.1 Determination Of An LQ-Controller For A High Velocity Flight Condition ..................................94
5.1.1 LQ-Controller Theory ................................................................. 94
5.1.2 Determination Of The Controller ................................................. 95
5.1.3 Simulation Of The Closed-Loop System With An Initial Roll Rate And Side Slip .............................................................................. 97
5.1.4 Extension To Command Directly A Roll Rate And A Side Slip .......... 100
5.1.5 Simulation Of A Directly Commanded Roll Rate And Side Slip Using The Extended Controller ......................................................... 101
5.1.6 Evaluation Of An Asymptotic Model Following ............................ 104
5.2 Determination Of An LQ-Controller For A Low Velocity Flight Condition ............................................................................................. 105
5.2.1 Design Defaults And Obtained Controller ...................................... 105
5.2.2 Simulation Of Initial Conditions For The Roll Rate And The Side Slip ......................................................................................... 106
5.2.3 Simulation Of Directly Commanded Impulses ............................... 108
5.3 Qualification Of The Developed LQ-Controller ............................... 111
6. Conclusion .......................................................................................... 112

Appendix
A Additional Plots .................................................................................... 114
A.3 Additional Plots For The Application Of The Eigenstructure Assignment ......................................................................................... 114
A.4 Additional Plots For The Gain Scheduling ........................................ 118
B Developed Matlab Files ...................................................................... 121
B.1 Files Used For The Tailless F-16 ....................................................... 121
B.1.1 Recent Files For The Trimming And Linearization Of The Model Using Matlab Version 5.1 ................................................................. 121
B.1.2 Linearized Data Of The Tailless F-16 .......................................... 126
B.1.3 Constants And Basic Data Used For The Original And Modified F-16 Model .................................................................................. 138
B.2 Eigenstructure Assignment .............................................................. 140
B.3 Placing The Eigenvalues Of The Desired Extended System On The Demanded Locations ................................................................. 144
B.4 Supporting Files ................................................................................ 158

Bibliography ........................................................................................... 162
Table Of Figures

Chapter 1. The Tailless F-16 Model

1.1 Evolution of the poles of the tailless F-16 for a variation of the velocity at a straight sea level flight. The velocity was varied from $V_t = 200$ [ft/s] — denoted by a “x” — to $V_t = 620$ [ft/s] — “o.” Page 23

Chapter 2. Introduction Of The Eigenstructure Assignment

2.1 State-space model of a dynamic system extended by a full state feedback loop with a gain matrix. Page 28

Chapter 3. Applying The Eigenstructure Assignment For A Lateral Control Design At Different Operating Points

3.1 Transfer functions for the desired system and the closed-loop system of the tailless aircraft which was achieved using the eigenstructure assignment. Page 39

3.2 Time history response of the desired and the achieved system to a commanded roll rate of $p_{cmd} = 1$ [$^\circ$/s]. Page 42

3.3 Time history response of the desired and the achieved system to a commanded side slip of $\beta_{cmd} = 1$ [$^\circ$]. Page 43

3.4 Closed-loop system of the tailless F-16 extended with an actuator model. Page 44

3.5 Dynamic system of the actuator model. Page 45

3.6 Desired system extended with the dynamic actuator model. Page 46

3.7 Time history response of the desired and the achieved extended system to a commanded roll rate of $p_{cmd} = 1$ [$^\circ$/s]. The break-off frequency of the actuators was chosen to $\tau_{act} = 20.202$ [rad/s]. Page 48

3.8 Actuator deflections of the closed-loop system for a commanded roll rate of $p_{cmd} = 1$ [$^\circ$/s]. Page 49

3.9 Actuator deflections of the closed-loop system for a commanded side slip of $\beta_{cmd} = 1$ [$^\circ$]. Page 50

3.10 Maximum deflection of the thrust vector $\epsilon_{tv}$ as a function of the Dutch roll frequency, $\omega_{dr}$, and damping, $\zeta_{dr}$. The Dutch roll damping was varied with a step size of 0.05 from 0.2 to 0.5. The dash-dotted line shows the limit of the actuator. Page 51

3.11 Deflections of the thrust vector for a commanded roll rate of $p_{cmd} = 1$ [$^\circ$/s]. While the position of the Dutch roll pole for the left plot was the original position with $\omega_{dr} = 2.0$ [rad/s], the value for the right plot was chosen to be $\omega_{dr} = 3.0$ [rad/s]. Page 53
3.12 Time history response of the side slip $\beta$ of the desired and the achieved system to a commanded roll rate of $p_{\text{cmd}} = 90 [^\circ/\text{s}]$. For the left plot the Dutch roll frequency is $\omega_{\text{dr}} = 2.0$ [rad/s] while it is $\omega_{\text{dr}} = 3.0$ [rad/s] for the right plot.  

3.13 Time history response of the desired and the achieved system to a commanded roll rate of $p_{\text{cmd}} = 1 [^\circ/\text{s}]$ at an operating point with a low velocity of $V_t = 200$ [ft/s].  

3.14 Time history response of the extended desired system and the achieved system to a commanded roll rate of $p_{\text{cmd}} = 1 [^\circ/\text{s}]$ at an operating point with a low velocity $V_t = 200$ [ft/s].  

3.15 Comparison of the desired transfer functions from the commanded roll rate to the side slip for two different velocities. For the lower velocity, the extended model was used.  

3.16 Positions of the Dutch roll pole and the numerator zero of the transfer function $p_{\text{cmd}}$ to $p$ for a variation of the Dutch roll frequency, $\omega_{\text{dr}}$, from 2 to 8 [rad/s] and a constant damping of $\zeta_{\text{dr}} = 0.25 [-]$. The dash-dotted lines connect corresponding points with the same $\omega_{\text{dr}}$ on both lines.  

3.17 Time history response of the desired extended system and the achieved system to a commanded roll rate of $p_{\text{cmd}} = 1 [^\circ/\text{s}]$ at an operating point with a low velocity $V_t = 200$ [ft/s]. The Dutch roll damping was increased to $\zeta_{\text{dr}} = 0.5 [-]$ while the frequency was kept at $\omega_{\text{dr}} = 2.0$ [rad/s].  

3.18 Time history response of the desired extended system and the achieved system to a commanded roll rate of $p_{\text{cmd}} = 1 [^\circ/\text{s}]$ at an operating point with a low velocity $V_t = 200$ [ft/s]. The Dutch roll frequency was increased to $\omega_{\text{dr}} = 6.0$ [rad/s] while the damping was kept at $\zeta_{\text{dr}} = 0.25 [-]$.  

3.19 Time history response of the desired extended system and the achieved system to a commanded roll rate of $p_{\text{cmd}} = 1 [^\circ/\text{s}]$ at an operating point with a low velocity $V_t = 200$ [ft/s]. The Dutch roll pole was determined with the frequency $\omega_{\text{dr}} = 2.5$ [rad/s] and the damping $\zeta_{\text{dr}} = 0.5 [-]$.  

3.20 Actuator deflections for a commanded roll rate of $p_{\text{cmd}} = 90 [^\circ/\text{s}]$ at the operating point with the velocity $V_t = 200$ [ft/s].  

3.21 Time history response of the extended system to a commanded side slip of $\beta_{\text{cmd}} = 1 [^\circ]$ at an operating point with a low velocity $V_t = 200$ [ft/s]. The position of the Dutch roll poles is the one determined in the last chapter.  

3.22 Actuator deflections for a commanded side slip of $\beta_{\text{cmd}} = 1 [^\circ]$ at the operating point with the velocity $V_t = 200$ [ft/s].  

Chapter 4. Approach For A Gain Scheduling Of The Closed-Loop System Of The Tailless F-16  

4.1 Entries of the first row of the gain matrix, $K$, as functions of the velocity, $V_t$. The dashed line shows a second-order polynomial that fits the data of the original entry in a least-square sense.
4.2 Entries of the second row of the gain matrix, \( K \), as functions of the velocity, \( V_t \). The dashed line shows a second-order polynomial that fits the data of the original entry in a least-square sense.

4.3 Entries of the transfer matrix, \( H \), as functions of the velocity, \( V_t \).

4.4 Maximum peak of the time history responses resulting from a simulation of a commanded roll rate of \( p_{cmd} = 1 \, [\text{°}/\text{s}] \) with the scheduled gain matrix. The solid line represents the original gain data and the dashed line the simplified one.

4.5 Maximum peak of the actuator deflections resulting from a simulation of a commanded roll rate of \( p_{cmd} = 1 \, [\text{°}] \) with the scheduled gain matrix. The solid line represents the original gain data and the dashed line the simplified one.

4.6 Maximum peak of the time history responses resulting from a simulation of a commanded roll rate of \( \beta_{cmd} = 1 \, [\text{°}] \) with the scheduled gain matrix. The solid line represents the original gain data and the dashed line the simplified one.

4.7 Maximum peak of the actuator deflections resulting from a simulation of a commanded roll rate of \( \beta_{cmd} = 1 \, [\text{°}] \) with the scheduled gain matrix. The solid line represents the original gain data and the dashed line the simplified one.

4.8 Time History Response of the closed-loop system for the flight condition with the velocity \( V_t = 300 \, [\text{ft/s}] \) to a commanded roll rate \( p_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.9 Actuator deflections of the closed-loop system for the flight condition with the velocity \( V_t = 300 \, [\text{ft/s}] \) to a commanded roll rate \( p_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.10 Time History Response of the closed-loop system for the flight condition with the velocity \( V_t = 500 \, [\text{ft/s}] \) to a commanded roll rate \( p_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.11 Actuator deflections of the closed-loop system for the flight condition with the velocity \( V_t = 500 \, [\text{ft/s}] \) to a commanded roll rate \( p_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.12 Time History Response of the closed-loop system for the flight condition with the velocity \( V_t = 300 \, [\text{ft/s}] \) to a commanded side slip \( \beta_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.13 Actuator deflections of the closed-loop system for the flight condition with the velocity \( V_t = 300 \, [\text{ft/s}] \) to a commanded side slip \( \beta_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.14 Time History Response of the closed-loop system for the flight condition with the velocity \( V_t = 500 \, [\text{ft/s}] \) to a commanded side slip \( \beta_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.

4.15 Actuator deflections of the closed-loop system for the flight condition with the velocity \( V_t = 500 \, [\text{ft/s}] \) to a commanded side slip \( \beta_{cmd} = 1 \, [\text{°}] \) using the gain scheduling.
Chapter 5. Development And Evaluation Of A Controller For The Tailless Aircraft Using LQ-Design

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Response of the closed-loop system to an initial roll rate of $p_{ini} = 1 , [\text{°/s}]$ at a flight condition with the velocity $V_t = 620 , [\text{ft/s}]$.</td>
<td>97</td>
</tr>
<tr>
<td>5.2</td>
<td>Response of the closed-loop system to an initial side slip of $\beta_{ini} = 1 , [\text{°}]$ at a flight condition with the velocity $V_t = 620 , [\text{ft/s}]$.</td>
<td>99</td>
</tr>
<tr>
<td>5.3</td>
<td>Basic approach of the system layout for the derivation of the transformation matrix $H$.</td>
<td>100</td>
</tr>
<tr>
<td>5.4</td>
<td>Response of the closed-loop system to a directly commanded roll rate of $p_{cmd} = 1 , [\text{°/s}]$ at a flight condition with the velocity $V_t = 620 , [\text{ft/s}]$.</td>
<td>102</td>
</tr>
<tr>
<td>5.5</td>
<td>Response of the closed-loop system to a directly commanded side slip of $\beta_{cmd} = 1 , [\text{°}]$ at a flight condition with the velocity $V_t = 620 , [\text{ft/s}]$.</td>
<td>103</td>
</tr>
<tr>
<td>5.6</td>
<td>System structure of the asymptotic model following method.</td>
<td>104</td>
</tr>
<tr>
<td>5.7</td>
<td>Response of the closed-loop system to an initial roll rate of $p_{ini} = 1 , [\text{°/s}]$ at a flight condition with the velocity $V_t = 200 , [\text{ft/s}]$.</td>
<td>106</td>
</tr>
<tr>
<td>5.8</td>
<td>Response of the closed-loop system to an initial side slip of $\beta_{ini} = 1 , [\text{°}]$ at a flight condition with the velocity $V_t = 200 , [\text{ft/s}]$.</td>
<td>107</td>
</tr>
<tr>
<td>5.9</td>
<td>Response of the closed-loop system to a directly commanded roll rate of $p_{cmd} = 1 , [\text{°/s}]$ at a flight condition with the velocity $V_t = 200 , [\text{ft/s}]$.</td>
<td>109</td>
</tr>
<tr>
<td>5.10</td>
<td>Response of the closed-loop system to a directly commanded side slip of $\beta_{cmd} = 1 , [\text{°}]$ at a flight condition with the velocity $V_t = 200 , [\text{ft/s}]$.</td>
<td>110</td>
</tr>
</tbody>
</table>
# Table Of Variables And Acronyms

This table is meant to give an overview of the most important variables and acronyms used in the report. For multiply used variables and acronyms either description is specified. The respective meaning then results from the context in which the denotation is used.

**Variables**

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<thead>
<tr>
<th>Variable</th>
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</tr>
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<td>A</td>
<td>system matrix</td>
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<tr>
<td>B</td>
<td>control matrix</td>
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<tr>
<td>C</td>
<td>output matrix</td>
</tr>
<tr>
<td>D</td>
<td>direct feed-through matrix</td>
</tr>
<tr>
<td>g</td>
<td>universal constant of gravitation</td>
</tr>
<tr>
<td>h</td>
<td>altitude</td>
</tr>
<tr>
<td>H</td>
<td>transformation matrix — Hamilton matrix</td>
</tr>
<tr>
<td>i</td>
<td>imaginary operator</td>
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<tr>
<td>I</td>
<td>identity matrix</td>
</tr>
<tr>
<td>K</td>
<td>gain matrix</td>
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<tr>
<td>Ma</td>
<td>Mach number</td>
</tr>
<tr>
<td>p</td>
<td>roll rate</td>
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<tr>
<td>P</td>
<td>power — weighting matrix for the eigenstructure assignment</td>
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<td>r</td>
<td>yaw rate</td>
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<tr>
<td>s</td>
<td>Laplace variable</td>
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<td>eigenvector</td>
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<tr>
<td>V</td>
<td>velocity — modal transformation matrix</td>
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<td>x</td>
<td>state vector</td>
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<td>y</td>
<td>output vector</td>
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<tr>
<td>α</td>
<td>angle of attack</td>
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<td>side slip angle</td>
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<td>δ</td>
<td>actuator deflection</td>
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<td>roll angle</td>
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<td>flight path angle</td>
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<tr>
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<td>throttle setting</td>
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<tr>
<td>λ</td>
<td>eigenvalue</td>
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<tr>
<td>μ</td>
<td>Lagrange multiplier</td>
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<tr>
<td>θ</td>
<td>pitch angle</td>
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<tr>
<td>σ</td>
<td>real part of a complex value</td>
</tr>
<tr>
<td>τ</td>
<td>break-off frequency</td>
</tr>
<tr>
<td>ω</td>
<td>frequency — imaginary part of a complex value</td>
</tr>
</tbody>
</table>
ψ yaw angle
ζ damping

Subscripts - Indices

a achievable/attainable
act actuator
aero aerodynamic
ail aileron
cg center of gravity
cl closed-loop
cmd commanded
d desired
dr Dutch roll
elv elevator
eng engine
ext extended
ini initial
max maximum
out output
sprl spiral
t true
tv thrust vector

0 trim point
1, 2, … numeration

Superscripts

T transposed
* conjugate complex transposed